

Strategies for Multiplication and Division Facts

Basic facts for multiplication are all those where both factors (the numbers being multiplied) are less than 10. The answer to a multiplication fact is called the product.

Basic division facts are all those that correspond to the multiplication facts. The answer to a division fact is called the quotient.

Multiplication is very much an extension of addition. 4×8 is thought of as 4 groups of 8. To get the answer, many students will add $8 + 8 + 8 + 8$ or draw pictures and count. For small numbers, this may be a reasonable strategy, but it is not at all efficient for larger numbers, so your goal is to help the student develop strategies that don't require a lot of counting.

All that is necessary for mastery of division facts is mastery of the multiplication facts and an understanding of the relationship between multiplication and division. When a student sees $48 \div 6$, he needs to ask, and be able to answer, the question '6 times what is 48'. Be sure to use both notations for division: $6 \div 2$ and $2 \overline{)6}$.

Work through one strategy at a time. Remember to stop and review (practice strategy selection) after a few have been learned.

- 1. Order of factors:** One concept the student needs to figure out quickly is that the *order of the factors does not matter*. However, you want the student to realize this on her own. Do not state it as rule that simply must be obeyed. If 4×8 is 4 groups of 8, it may not be immediately clear to the student that 8×4 , which would be 8 groups of 4, is exactly the same. A rectangular array is a powerful tool for helping a student visualize this. Take some counters, and line them up into 4 rows (groups) of 8 counters each, and arrange these into a 4 by 8 rectangle. Now you have a visual representation that 4 rows (groups) of 8 is the same number of counters as 8 columns (groups) of 4. Knowing this reduces the number of multiplication facts by nearly half.
- 2. Doubles:** Facts that have 2 as a factor are equivalent to the addition doubles and should be easily known. The same visual pictures used for the addition doubles will work here, if needed. (2 rows of 6 eggs make a dozen, 2 hands with 5 fingers each make 10 fingers, etc.)
- 3. Fives and other skip counting:** Facts that have 5 as a factor require counting by 5's. You can use rectangular arrays that have rows of 5 dots or even relate these to the minute hand of a clock. Most students can, and should, be able to count by 5's. Practice, if needed. *Whether this strategy will be useful to the student for other facts depends on the student's ability to skip count. You'll have to check this before proceeding. Other strategies may be more effective for a particular student. To get 3×4 using skip counting, the student might do the following: Count by 4 three times: 4, 8, 12. Note this is still counting 3 groups of 4.*
- 4. Zeros and Ones:** Though easy to an adult, these facts can often cause trouble for students. Why is it that $6 + 0$ is 6, but 6×0 is 0? Or, if $1 + 5$ is 6, why is 1×5 still 5? Use word problems and pictures/counters to help the student visualize and develop his own reasoning for this. (What would 4 groups of 0 look like? Or 1 group of 4?) Try to avoid simply stating a rule to be memorized.

5. **Nines:** There are many ‘easy’ ways to get the nines facts. You might try having the student write down all the nines facts with the answers and look for patterns. They should notice that the sum of the digits in the answer is always 9 and also that the first digit in the answer is one less than the other factor (the one that isn’t 9). These two observations can be used together to get any nines fact. An alternative strategy is to recognize that, for example, 6×9 is 6×10 less one set of 6, or $60 - 6$. Many students find this subtraction quite easy and prefer this strategy.
6. **Helping Facts:** The remaining facts can be related to already known facts, usually in several different ways. Which facts to use for help depends on the factors, which factor you focus on, and the student’s ability to do mental addition. Here are some examples of using helping facts:

Facts with a 4 as a factor: *Double and then double again.* For example, 4×6 : double 6 to get 12 and then double this to get 24.

Facts with a 3 as a factor: *Double and one more set.* For example, 3×6 : double 6 to get 12 and then add 6 more to get 18.

Facts with an even factor: *Half then double.* For example, 6×7 : Take half of 6 which is 3 and multiply by 7 to get 21; then double this to get 42. (The helping fact 3×7 must be known for this to work quickly.)

Get close and add one more set: Suppose you want to know 6×7 . This is six 7’s. It’s the same as five 7’s (5×7) plus one more 7.

What about the ‘hard’ fact 7×8 ? Try this: You need seven 8’s, so start with five 8’s (40) and add two more 8’s (16) to get 56. *This can be used with any fact where at least one factor is greater than 5. The goal is to break one factor into 5 and some more, as in this example.*

7. Mastery of **division** facts depends solely on the student’s understanding of the relationship between multiplication and division and his mastery of the multiplication facts.

Fact families should be discussed, modeled, and written. An example is

$$3 \times 5 = 15 \quad 5 \times 3 = 15 \quad 15 \div 3 = 5 \quad 15 \div 5 = 3$$

Multiplication Facts list through 9 x 9

Use to check for mastery (you call out facts randomly) and/or to keep track of mastered facts.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Division Facts List

Use to check for mastery (you call out facts randomly) and/or to keep track of mastered facts.
Notice these correspond to the multiplication facts through 9×9 .

$0 \div 1 = 0$	$0 \div 2 = 0$	$0 \div 3 = 0$	$0 \div 4 = 0$
$1 \div 1 = 1$	$2 \div 2 = 1$	$3 \div 3 = 1$	$4 \div 4 = 1$
$2 \div 1 = 2$	$4 \div 2 = 2$	$6 \div 3 = 2$	$8 \div 4 = 2$
$3 \div 1 = 3$	$6 \div 2 = 3$	$9 \div 3 = 3$	$12 \div 4 = 3$
$4 \div 1 = 4$	$8 \div 2 = 4$	$12 \div 3 = 4$	$16 \div 4 = 4$
$5 \div 1 = 5$	$10 \div 2 = 5$	$15 \div 3 = 5$	$20 \div 4 = 5$
$6 \div 1 = 6$	$12 \div 2 = 6$	$18 \div 3 = 6$	$24 \div 4 = 6$
$7 \div 1 = 7$	$14 \div 2 = 7$	$21 \div 3 = 7$	$28 \div 4 = 7$
$8 \div 1 = 8$	$16 \div 2 = 8$	$24 \div 3 = 8$	$32 \div 4 = 8$
$9 \div 1 = 9$	$18 \div 2 = 9$	$27 \div 3 = 9$	$36 \div 4 = 9$

$0 \div 5 = 0$	$0 \div 6 = 0$	$0 \div 7 = 0$	$0 \div 8 = 0$
$5 \div 5 = 1$	$6 \div 6 = 1$	$7 \div 7 = 1$	$8 \div 8 = 1$
$10 \div 5 = 2$	$12 \div 6 = 2$	$14 \div 7 = 2$	$16 \div 8 = 2$
$15 \div 5 = 3$	$18 \div 6 = 3$	$21 \div 7 = 3$	$24 \div 8 = 3$
$20 \div 5 = 4$	$24 \div 6 = 4$	$28 \div 7 = 4$	$32 \div 8 = 4$
$25 \div 5 = 5$	$30 \div 6 = 5$	$35 \div 7 = 5$	$40 \div 8 = 5$
$30 \div 5 = 6$	$36 \div 6 = 6$	$42 \div 7 = 6$	$48 \div 8 = 6$
$35 \div 5 = 7$	$42 \div 6 = 7$	$49 \div 7 = 7$	$56 \div 8 = 7$
$40 \div 5 = 8$	$48 \div 6 = 8$	$56 \div 7 = 8$	$64 \div 8 = 8$
$45 \div 5 = 9$	$54 \div 6 = 9$	$63 \div 7 = 9$	$72 \div 8 = 9$

$0 \div 8 = 0$	$0 \div 9 = 0$
$8 \div 8 = 1$	$9 \div 9 = 1$
$16 \div 8 = 2$	$18 \div 9 = 2$
$24 \div 8 = 3$	$27 \div 9 = 3$
$32 \div 8 = 4$	$36 \div 9 = 4$
$40 \div 8 = 5$	$45 \div 9 = 5$
$48 \div 8 = 6$	$54 \div 9 = 6$
$56 \div 8 = 7$	$63 \div 9 = 7$
$64 \div 8 = 8$	$72 \div 9 = 8$
$72 \div 8 = 9$	$81 \div 9 = 9$